Complex Numbers Operations

Department of Electrical Engineering

Outline

- Complex numbers:
  - Addition
  - Subtraction
  - Multiplication
  - Division
- Conversion of complex numbers (Cartesian & polar coordinates)
- Introduction to Matlab

Definition

- A complex number is a combination of a :
  - Real number

    12, 4.6, ¾, …. Any number you can think of !
  - Imaginary number

    Special numbers because … imaginary^2 → negative
- The "unit" imaginary number is i, like 1 for real numbers.

$$i = \sqrt{-1}$$

- by simply **accepting** that **i** exists we can solve things that need the square root of a negative number.

DEFINITION

- A Complex Number is a combination of a Real Number and an Imaginary Number

- Examples :
  -

1 + i	39 + 3i	0.8 - 2.2i	-2 + πi	√2 + i/2

COMPLEX NUMBER

- So, a Complex Number has a real part and an imaginary part. But either part can be **0**, so all Real Numbers and Imaginary Numbers are also Complex Numbers.

Complex Number	Real Part	Imaginary Part
3 + 2i	3	2
5	5	0
-6i	0	-6

Addition of complex numbers

- To add two complex numbers we add each element separately:

$$(a+b\mathbf{i}) + (c+d\mathbf{i}) = (a+c) + (b+d)\mathbf{i}$$

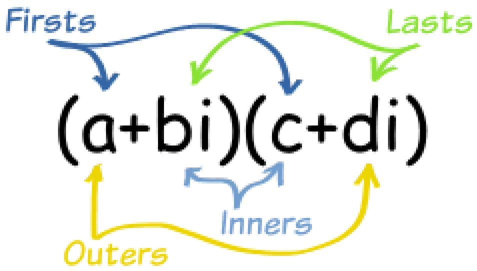
- Example:

$$(3 + 2i) + (1 + 7i) = (4 + 9i)$$

- Subtraction follows the same rule !

MULTIPLICATION

- **Each part of the first complex number** gets multiplied by **each part of the second complex number**



$(a+b\boldsymbol{i})(c+d\boldsymbol{i}) = ac + ad\boldsymbol{i} + bc\boldsymbol{i} + bd\boldsymbol{i}^2$

Example

$(3 + 2i)(1 + 7i) = 3\times1 + 3\times7i + 2i\times1 + 2i\times7i$

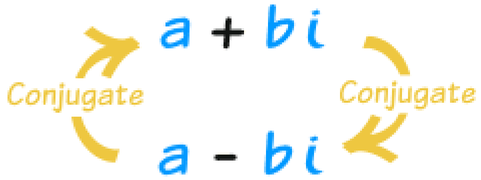
$= 3 + 21i + 2i + 14i^2$

$= 3 + 21i + 2i - 14$ (because $i^2 = -1$)

$= -11 + 23i$

Division

- The trick is to **multiply both top and bottom** by the **conjugate of the bottom**.
- A conjugate is where you **change the sign in the middle** like this:



Example

Example: Do this Division:

$$\frac{2 + 3i}{4 - 5i}$$

Multiply top and bottom by the conjugate of 4 - 5i :

$$\frac{2 + 3i}{4 - 5i} \times \frac{4 + 5i}{4 + 5i} = \frac{8 + 10i + 12i + 15i^2}{16 + 20i - 20i - 25i^2}$$

Now remember that $i^2 = -1$, so:

$$= \frac{8 + 10i + 12i - 15}{16 + 20i - 20i + 25}$$

Add Like Terms (and notice how on the bottom 20i - 20i cancels out!):

$$= \frac{-7 + 22i}{41}$$

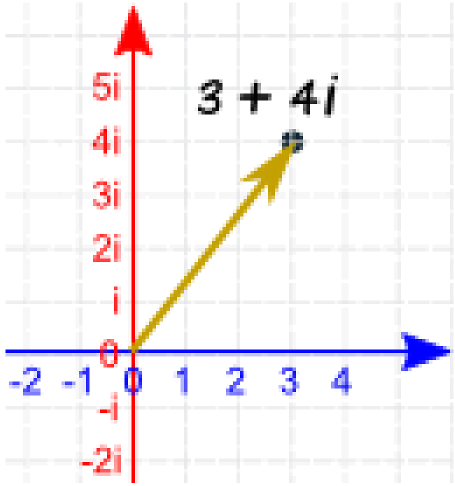
We should then put the answer back into a + bi form:

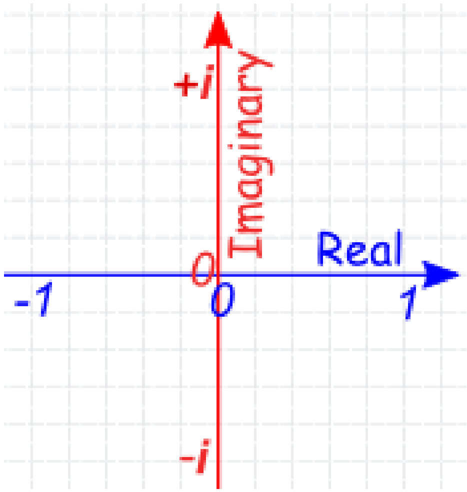
$$= \frac{-7}{41} + \frac{22}{41}i$$

Complex plane

- The Real part goes left-right
- The Imaginary part goes up-down
- Example

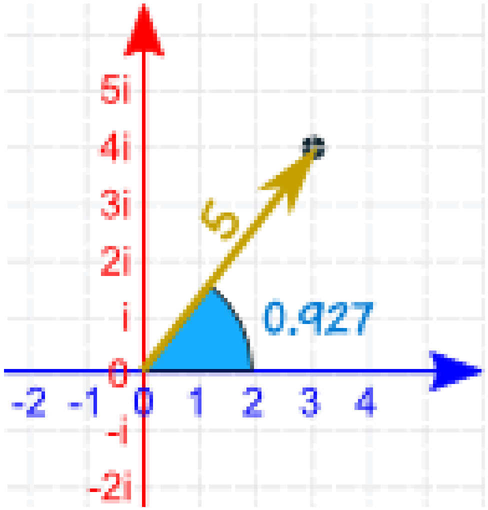
  $\mathbf{3 + 4}\boldsymbol{i}$





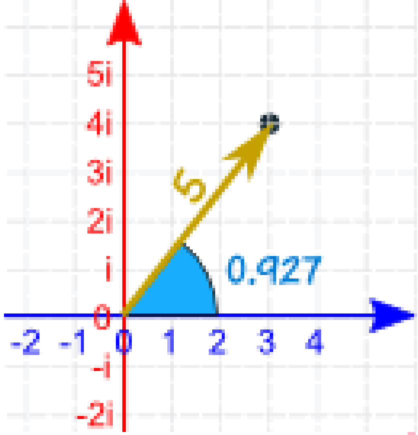
Polar plane

- the complex number **3 + 4*i*** can also be distance (5) and angle (0.927 radians).



- How to do the conversion ?

Conversion of complex numbers



- Example: the number **3 + 4i**

We can do a Cartesian to Polar conversion:

- $\mathbf{r = \sqrt{(x^2 + y^2)}} = \sqrt{(3^2 + 4^2)} = \sqrt{25} = \mathbf{5}$
- $\boldsymbol{\theta} = \mathbf{tan^{-1}\ (y/x)} = \tan^{-1}(4/3) = \mathbf{0.927}$ (to 3 decimals)

We can also take Polar coordinates and convert them to Cartesian coordinates:

- $\mathbf{x = r \times cos(\ \boldsymbol{\theta}\ )} = 5 \times \cos(\ 0.927\ ) = 5 \times 0.6002... = \mathbf{3}$ (close enough)
- $\mathbf{y = r \times sin(\ \boldsymbol{\theta}\ )} = 5 \times \sin(\ 0.927\ ) = 5 \times 0.7998... = \mathbf{4}$ (close enough)

Introduction to Matlab

References

- http://www.mathsisfun.com/numbers/complex-numbers.html

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