



COMPLEX NUMBERS OPERATIONS

Department of Electrical Engineering

OUTLINE

- Complex numbers:
 - Addition
 - Subtraction
 - Multiplication
 - Division
- Conversion of complex numbers (Cartesian & polar coordinates)
- Introduction to Matlab

DEFINITION

- A complex number is a combination of a :
 - Real number
12, 4.6, $\frac{3}{4}$, Any number you can think of !
 - Imaginary number
Special numbers because ... *imaginary*² \rightarrow *negative*
- The “unit” imaginary number is i , like 1 for real numbers.

$$i = \sqrt{-1}$$

- by simply **accepting** that i exists we can solve things that need the square root of a negative number.

DEFINITION

- A Complex Number is a combination of a Real Number and an Imaginary Number

The diagram shows the expression $a + bi$ in a stylized font. The letter a is blue, the plus sign is black, the letter b is yellow, and the letter i is blue. A blue wavy arrow points from the text "Real Part" to the a . A yellow wavy arrow points from the text "Imaginary Part" to the b . A blue wavy arrow points from the text " $\sqrt{-1}$ " to the i .

- Examples :

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$1 + i$	$39 + 3i$	$0.8 - 2.2i$	$-2 + \pi i$	$\sqrt{2} + i/2$
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COMPLEX NUMBER

- So, a Complex Number has a real part and an imaginary part. But either part can be **0**, so all Real Numbers and Imaginary Numbers are also Complex Numbers.

Complex Number	Real Part	Imaginary Part
$3 + 2i$	3	2
5	5	0
$-6i$	0	-6

ADDITION OF COMPLEX NUMBERS

- To add two complex numbers we add each element separately:

$$(a+bi) + (c+di) = (a+c) + (b+d)i$$

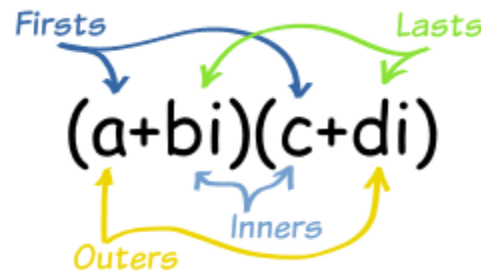
- Example:

$$(3 + 2i) + (1 + 7i) = (4 + 9i)$$

- Subtraction follows the same rule !

MULTIPLICATION

- **Each part of the first complex number gets multiplied by each part of the second complex number**



$$(a+bi)(c+di) = ac + adi + bci + bdi^2$$

EXAMPLE

$$(3 + 2i)(1 + 7i)$$

$$= 3 \times 1 + 3 \times 7i + 2i \times 1 + 2i \times 7i$$

$$= 3 + 21i + 2i + 14i^2$$

$$= 3 + 21i + 2i - 14$$

(because $i^2 = -1$)

$$= -11 + 23i$$

DIVISION

- The trick is to **multiply both top and bottom** by the **conjugate of the bottom**.
- A conjugate is where you **change the sign in the middle** like this:

The diagram illustrates the concept of a conjugate. It shows two complex numbers, $a + bi$ and $a - bi$, arranged vertically. The top number is $a + bi$ and the bottom number is $a - bi$. Two yellow curved arrows point from each number towards the other, with the word "Conjugate" written in yellow next to each arrow, indicating that they are conjugates of each other.

EXAMPLE

Example: Do this Division:

$$\frac{2 + 3i}{4 - 5i}$$

Multiply top and bottom by the conjugate of $4 - 5i$:

$$\frac{2 + 3i}{4 - 5i} \times \frac{4 + 5i}{4 + 5i} = \frac{8 + 10i + 12i + 15i^2}{16 + 20i - 20i - 25i^2}$$

Now remember that $i^2 = -1$, so:

$$= \frac{8 + 10i + 12i - 15}{16 + 20i - 20i + 25}$$

Add Like Terms (and notice how on the bottom $20i - 20i$ cancels out!):

$$= \frac{-7 + 22i}{41}$$

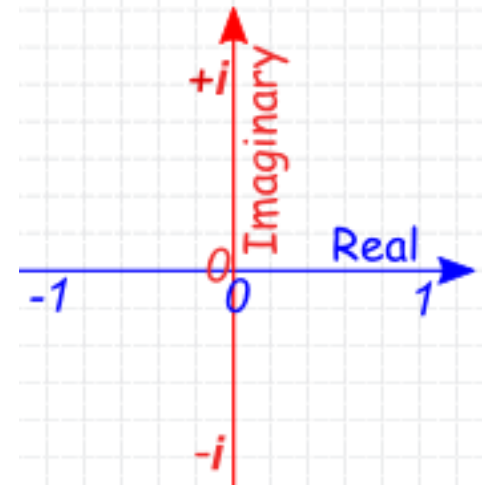
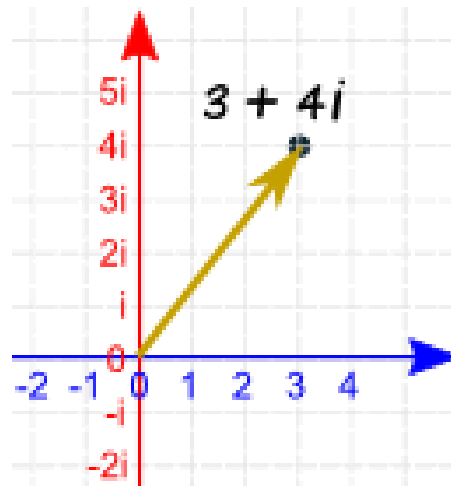
We should then put the answer back into $a + bi$ form:

$$= \frac{-7}{41} + \frac{22}{41}i$$

COMPLEX PLANE

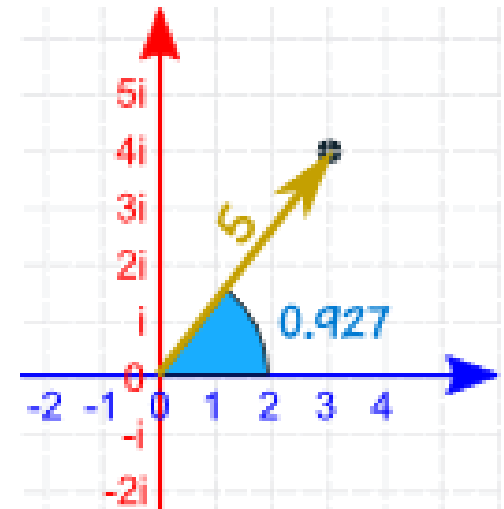
- The Real part goes left-right
- The Imaginary part goes up-down
- Example

$$3 + 4i$$



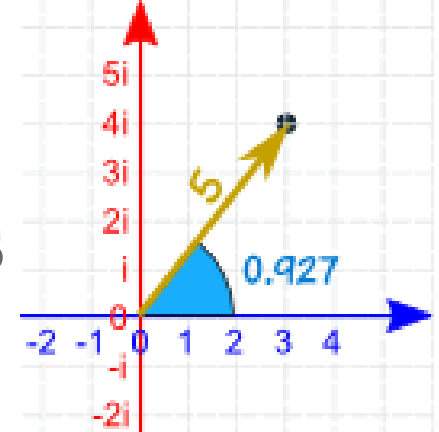
POLAR PLANE

- the complex number $3 + 4i$ can also be distance (5) and angle (0.927 radians).
- How to do the conversion ?



CONVERSION OF COMPLEX NUMBERS

- Example: the number $3 + 4i$



We can do a [Cartesian to Polar conversion](#):

- $r = \sqrt{(x^2 + y^2)} = \sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$
- $\theta = \tan^{-1}(y/x) = \tan^{-1}(4/3) = \mathbf{0.927}$ (to 3 decimals)

We can also take Polar coordinates and convert them to Cartesian coordinates:

- $x = r \times \cos(\theta) = 5 \times \cos(0.927) = 5 \times 0.6002... = 3$ (close enough)
- $y = r \times \sin(\theta) = 5 \times \sin(0.927) = 5 \times 0.7998... = 4$ (close enough)

INTRODUCTION TO MATLAB



REFERENCES

- <http://www.mathsisfun.com/numbers/complex-numbers.html>

